Valeurs exactes trigo

Pour calculer les sinus et cosinus de $\frac{\pi}{60}$ on part des valeurs classiques en remarquant que : $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$. Ceci permet de calculer sinus et cosinus des multiples de $\frac{\pi}{12}$.

On utilise ensuite l'équation $2\cos\frac{4\pi}{5} + 2\cos\frac{2\pi}{5} + 1 = 0$ pour déterminer le sinus et le cosinus des multiples de $\frac{\pi}{10}$, avec $\frac{\pi}{10} = \frac{\pi}{2} - \frac{2\pi}{5}$.

Ensuite

$$\frac{\pi}{60} = \frac{6\pi}{60} - \frac{5\pi}{60} = \frac{\pi}{10} - \frac{\pi}{12} \qquad \frac{\pi}{30} = \frac{6\pi}{30} - \frac{5\pi}{30} = \frac{\pi}{5} - \frac{\pi}{6} \qquad \frac{\pi}{20} = \frac{5\pi}{20} - \frac{4\pi}{20} = \frac{\pi}{4} - \frac{\pi}{5}$$

α	$\cos\alpha$	$\sin\alpha$	
$\frac{\pi}{60}$	$\frac{\sqrt{2}(\sqrt{3}-1)(\sqrt{5}-1)+2(\sqrt{3}+1)\sqrt{5+\sqrt{5}}}{16}$	$\frac{\sqrt{2}(\sqrt{3}+1)(\sqrt{5}-1)-2(\sqrt{3}-1)\sqrt{5+\sqrt{5}}}{16}$	$\frac{29\pi}{60}$
$\frac{\pi}{30}$	$\frac{\sqrt{3}(\sqrt{5}+1)+\sqrt{2}\sqrt{5-\sqrt{5}}}{8}$	$\frac{-1-\sqrt{5}+\sqrt{6}\sqrt{5-\sqrt{5}}}{8}$	$\frac{14\pi}{30}$
$\frac{\pi}{20}$	$\frac{\sqrt{2}(1+\sqrt{5})+2\sqrt{5-\sqrt{5}}}{8}$	$\frac{\sqrt{2}(1+\sqrt{5})-2\sqrt{5-\sqrt{5}}}{8}$	$\frac{9\pi}{20}$
$\frac{\pi}{15}$	$\frac{-1+\sqrt{5}+\sqrt{6}\sqrt{5+\sqrt{5}}}{8}$	$\frac{\sqrt{3}(1-\sqrt{5})+\sqrt{2}\sqrt{5+\sqrt{5}}}{8}$	$\frac{13\pi}{30}$
$\frac{\pi}{12}$	$\frac{\sqrt{2}(\sqrt{3}+1)}{4}$	$\frac{\sqrt{2}(\sqrt{3}-1)}{4}$	$\frac{5\pi}{12}$
$\frac{\pi}{10}$	$\frac{\sqrt{2}\sqrt{5+\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\frac{2\pi}{5}$
$\frac{7\pi}{60}$	$\frac{\sqrt{2}(\sqrt{3}+1)(\sqrt{5}+1)+2(\sqrt{3}-1)\sqrt{5-\sqrt{5}}}{16}$	$\frac{2(\sqrt{3}+1)\sqrt{5-\sqrt{5}}-\sqrt{2}(\sqrt{3}-1)(\sqrt{5}+1)}{16}$	$\frac{23\pi}{60}$
$\frac{2\pi}{15}$	$\frac{(\sqrt{5}+1)+\sqrt{6}\sqrt{5-\sqrt{5}}}{8}$	$\frac{\sqrt{3}(\sqrt{5}+1)-\sqrt{2}\sqrt{5-\sqrt{5}}}{8}$	$\frac{11\pi}{30}$
$\frac{3\pi}{20}$	$\frac{\sqrt{2}(\sqrt{5}-1)+2\sqrt{5+\sqrt{5}}}{8}$	$\frac{-\sqrt{2}(\sqrt{5}-1)+2\sqrt{5+\sqrt{5}}}{8}$	$\frac{7\pi}{20}$
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\pi}{3}$
$\frac{11\pi}{60}$	$\frac{2(\sqrt{3}+1)\sqrt{5+\sqrt{5}}-\sqrt{2}(\sqrt{3}-1)(\sqrt{5}-1)}{16}$	$\frac{\sqrt{2}(\sqrt{3}+1)(\sqrt{5}-1)+2(\sqrt{3}-1)\sqrt{5+\sqrt{5}}}{16}$	$\frac{19\pi}{60}$
$\frac{\pi}{5}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{2}\sqrt{5-\sqrt{5}}}{4}$	$\frac{3\pi}{10}$
$\frac{13\pi}{60}$	$\frac{\sqrt{2}(\sqrt{3}-1)(\sqrt{5}+1)+2(\sqrt{3}+1)\sqrt{5-\sqrt{5}}}{16}$	$\frac{\sqrt{2}(\sqrt{3}+1)(\sqrt{5}+1)-2(\sqrt{3}-1)\sqrt{5-\sqrt{5}}}{16}$	$\frac{17\pi}{60}$
$\frac{7\pi}{30}$	$\frac{\sqrt{3}(\sqrt{5}-1)+\sqrt{2}\sqrt{5+\sqrt{5}}}{8}$	$\frac{\sqrt{6}\sqrt{5+\sqrt{5}}-(\sqrt{5}-1)}{8}$	$\frac{4\pi}{15}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\pi}{4}$
	$\sin\beta$	$\cos\beta$	β